Reexamination and Correction of the Critical Radius for Radial Heat Conduction

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The purpose of this communication is to derive a more realistic expression for the critical radius, taking account of the dependence of the convective heat transfer coefficient on radius and on temperature difference as expressed by $h \sim r_0^{-m}|T_0 - T_\infty|^n$, m and $n \geqq 0$

For a cylinder transferring heat by convection from its external surface to a fluid environment, one can write

$$Q = 2\pi r_0 Lh\,(T_0 - T_\infty) \sim r_0^{1-m}\,(T_0 - T_\infty)\,|T_0 - T_\infty|^n \tag{1}$$

so that the operation $\partial Q/\partial r_0 = 0$, corresponding to an extremum for Q, yields

$$\partial T_0/\partial r_0 = -\,[(1-m)/(1+n)]\,(T_0 - T_\infty)/r_0^* \tag{2}$$

It can be shown that $\partial T_0/\partial r_0 = (\partial T/\partial r)_0$, and since $(\partial T/\partial r)_0 = -(h/k)(T_0 - T_\infty)$, Equation (2) gives

$$r_0^* = [(1-m)/(1+n)]\;k/h \tag{3}$$

The conventional result for the critical radius is $r_0^* = k/h$. Thus, the quantity $(1-m)/(1+n)$ is a correction factor ($\leqq 1$) accounting for the r_0 and ΔT dependences of h. As an example, consider forced convection flow across a cylinder. For N_{Re} (based on diameter) between 4,000 and 40,000, $m = 0.382$, while $n = 0$. Correspondingly, the correction factor $(1-m)/(1+n) = 0.618$. For a second example, consider free convection about a horizontal cylinder for which $m = n = 1/4$, giving $(1-m)/(1+n) = 0.6$.

A derivation for radial heat flow in a sphere, paralleling that given above, yields a critical radius r_0^* as follows:

$$r_0^* = [(1 - \tfrac{1}{2}m)/(1+n)]\;2k/h \tag{4}$$

Since the conventional expression for r_0^* for a sphere is $2k/h$, the quantity $(1-\tfrac{1}{2}m)/(1+n)$ is a correction factor.

NOTATION

- h = convective heat transfer coefficient
- k = thermal conductivity of solid adjacent to external surface
- L = cylinder length
- m = exponent of radius dependence
- n = exponent of temperature difference dependence
- Q = heat transfer rate at external surface
- r = radial coordinate
- r_0 = radius of external surface
- r_0^* = critical radius, corresponding to $\partial Q/\partial r_0 = 0$
- T = temperature
- T_0 = temperature of external surface
- T_∞ = fluid temperature

Analysis of Steady State Shearing and Stress Relaxation in the Maxwell Orthogonal Rheometer: Corrigenda and Addenda

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We have recently noted a similarity between the material functions in Equations (18) and (19) of the above article [*AIChE J.*, 14, 758-761 (1968)] and the material functions obtained for large deformation steady shear flow with superposed infinitesimal-amplitude transverse oscillatory motion.

Consider the flow with velocity components

$$v_1 = Kx_3 \qquad v_2 = x_3 Re\,\{A\;e^{i\omega t}\} \qquad v_3 = 0 \tag{1}$$

where K is the velocity gradient associated with the steady shear flow in the x_1 direction and A is a complex quantity which gives the amplitude and phase of the oscillatory motion in the x_2 direction. The displacement functions are then

$$x_1 = x'_1 + Kx'_3(t - t') \qquad x_2 = x_2' + x'_3 Re\,\{(A/i\omega)(e^{i\omega t} - e^{i\omega t'})\} \qquad x_3 = x'_3 \tag{2}$$

For this flow pattern, one obtains for the same rheological model [Equations (1) to (4) of our original paper]:

$$\tau_{23} = -\,Re\left\{A \sum_{p=1}^{\infty} \frac{\eta_p\;e^{i\omega t}}{(1+\lambda^2_{1p}K^2)(1+i\lambda_{2p}\omega)}\right\} \tag{3}$$

If we write $\tau_{23} = Re\,\{\tau^0_{23}\,e^{i\omega t}\}$ with $\tau^0_{23} = -\,(\eta'_\perp - i\eta''_\perp)A$, then we obtain the material functions

$$\eta'_\perp = \sum_{p=1}^{\infty} \frac{\eta_p}{(1+\lambda^2_{1p}K^2)(1+\lambda^2_{2p}\omega^2)} \tag{4}$$

$$\eta''_\perp = \sum_{p=1}^{\infty} \frac{\eta_p(\lambda_{2p}\omega)}{(1+\lambda^2_{1p}K^2)(1+\lambda^2_{2p}\omega^2)} \tag{5}$$

Note that Equations (4) and (5) are the same as the expressions for $\tau_{xz}(-\Omega\psi)$ and $\tau_{yz}/(-\Omega\psi)$ of Equations (18) and (19) of our publication, if K is replaced by $\Omega\psi$ and ω by Ω.

The equivalence of the material functions $\eta'_\perp$ and $\eta''_\perp$ with the Maxwell orthogonal rheometer functions is probably fortuitous, since the strain histories [that is, $\Gamma_{ij}(t, t')$] for the two flows are unequal. This equivalence may, in fact, by symptomatic of a shortcoming of the rheological model used.

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